Application of the Modified Bartz Analysis to Very Large Nozzles

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THE advent of large, solid propellant boosters has created a need for design information on convective heat transfer and boundary-layer growth in very large nozzles. The convective heat transfer in converging-diverging nozzles is usually computed either from empirical equations obtained from the correlation of experimental heat-transfer data or integral boundary-layer analyses. The difficulty at present is that no experimental data are available for heat transfer in very large nozzles. Therefore, the accuracy of the various methods has not been determined. However, recent results of Back, Massier, and Gier[1] show that the predictions of the modified Bartz analysis,[2] with physical properties evaluated at the film temperature, give better agreement with their experimental data than other methods, although these data are for a relatively small nozzle. Moreover, the modified Bartz analysis has been programed for automatic computation. The ability of the modified Bartz analysis to predict convective heat transfer reasonably accurately in small nozzles and the fundamental basis of this analysis recommend its use in large nozzles.

The applicability of the modified Bartz analysis to very large nozzles has not yet been established. Indeed, the experimental heat-transfer results employed to test the analysis were obtained in nozzles approximately 50 times smaller than a space-booster nozzle. The objective of this note is to report the results of a short study of the applicability of the modified Bartz analysis for predicting boundary-layer characteristics and heat transfer in a very large nozzle. In this study, the velocity boundary-layer thickness and displacement thickness predicted by the modified Bartz analysis were compared with the measured thicknesses of the boundary layer from the 16-ft supersonic wind tunnel at Arnold Engineering Development Center (AEDC).‡ Although no heat-transfer data were available for the comparison, it is believed that the comparison is still of value since convective heat transfer is intimately connected with the thickness of the boundary layer.

The nozzle in the tunnel is two-dimensional, the side walls are contoured, and the ceiling and floor are flat. The boundary-layer measurements in the tunnel were made 52.00 ft from the throat of the nozzle on the centerline of the ceiling of the tunnel. Data were recorded for exit Mach numbers of 1.5, 1.6, 1.75, and 2.0. However, since the exit Mach numbers of large booster rocket nozzles are on the order of 3.0, only the data from the Mach 2.0 test were selected for the comparison. For this test, the stagnation temperature was 750°R, and the stagnation pressure was 1000 psfa. The temperature at the tunnel wall and the temperature distribution through the thermal boundary layer were not recorded during the boundary-layer measurements at AEDC.

The modified Bartz analysis, which in general applies to axisymmetric geometry, was adapted to the two-dimensional tunnel geometry by assuming a very large, constant radius. The wall temperature was taken to be 700°R, and the free-stream pressure gradient was based on the results of one-dimensional theory. Since no initial boundary-layer thicknesses were measured, the momentum thickness at the throat, θ_i, was varied from 0.001 to 0.044 in. to determine the sensitivity of the theoretical results to the initial momentum thickness.

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Table 1 Comparison of theoretical and experimental results

	Modified Bartz analysis		AEDC data
Momentum thickness at the throat, in.	0.001	0.044	...
Thickness of the velocity boundary layer, in., 52.00 ft from the throat	6.96	7.06	6.10
Thickness of the thermal boundary layer, in., 52.00 ft from the throat	5.49	6.09	no data
Displacement thickness, in., 52.00 ft from the throat	1.63	1.64	1.49

The theoretical and experimental results are presented in Table 1. The table shows that the theoretical conditions at the nozzle exit are relatively insensitive to the value selected for the momentum thickness at the nozzle throat. Table 1 shows also that the theoretical and experimental results are in reasonable agreement. There are two probable causes for the slight disagreement. First, the boundary layer on the ceiling of the tunnel is not strictly two-dimensional, since a small pressure gradient exists across the ceiling because of the curved side walls. This pressure gradient will induce a secondary flow from the center of the tunnel toward the side walls. This flow should cause a reduction in the boundary-layer thickness from that for strictly two-dimensional flow. The theoretical and experimental data show this trend. Second, the experimentally obtained velocity profiles demonstrate good agreement with a $\frac{1}{9}$ power law, rather than the $\frac{1}{7}$ power law employed in the modified Bartz analysis.

Although the comparison was limited to boundary-layer data for a single condition, the results are encouraging and give some justification for the use of the modified Bartz analysis in very large nozzles.

References

[1] Back, L. H., Massier, P. F., and Gier, L. H., "Convective heat transfer in a convergent-divergent nozzle," Intern. J. Heat Mass Transfer **7**, 549 (1964).

[2] Elliot, D. G., Bartz, D. R., and Silver, S., "Calculations of turbulent boundary layer growth and heat transfer in axisymmetric nozzles," Jet Propulsion Lab., TR 32-387 (February 1963).

A Method of Estimating the Effect of Shock Interaction on Stagnation Line Heating

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Nomenclature

a = acoustic velocity
D = $1 - k/[k(2-k)]^{1/2}$
d = diameter of leading edge
E = $\sinh^{-1} D$
k = $2\rho_1/(\rho_2 + \rho_{2t})$
M = Mach number
T = absolute temperature
u = velocity component in x direction
V = velocity vector
v = velocity component in y direction
x = coordinate
y = coordinate

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